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METHODUS FIGURARUM LINEIS RECTIS & CURVIS COMPREHENSARUM QUADRATURAS DETERMINANDI.

Authore JOHANNES CRAIGE.

LONDINI:

Impenſis *Moſis Pitt*, ad inſigne Angeli in Cœmeterio D. *Paulo*, MDCLXXXV.

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HONORABILI VIRO

DOMINO

ROBERTO DAWES,

BARONETTO

ANGLO,

TRACTATUM HUNC,

Benevolentiæ & Obſervantiæ ergo,

D. C. Q.

JOHANNES CRAIGE.

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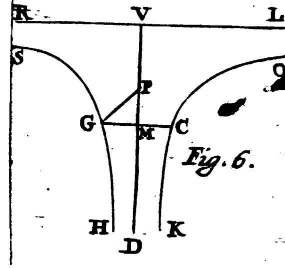


Fig. 6.

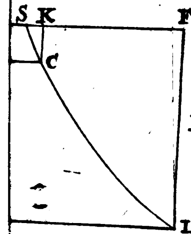


Fig. 11.

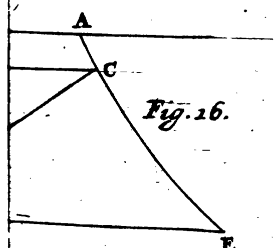


Fig. 16.

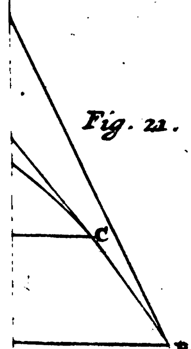


Fig. 21.

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Methodus Figurarum, &c.

OPtimè nuper obſervârunt Geometræ quaſdam eſſe Figuras indefinitæ Quadraturæ capaces, quæ tum quoad totas, tum quoad ſingulas partes ſunt Quadrabiles; Alias vero eſſe, qnæ, licet hujuſmodi Quadraturam indefinitam non admittant, aliquam tamen habent portionem quadrabilem, imo tota figura nonnunquam Quadrari poteſt, cum quælibet ejus pars non poſſit. Nec credibile eſt ex alio fonte eornm errorem ortum eſſe, qui Circuli, Hyperbolæ, & aliarum quarundam figurarum Quadraturas impoſſibiles exiſtimârunt, quam quia hanc figurarum diſtinctionem non conſiderârunt. Methodis enim utentes quæ ſupponunt figuras eſſe indefinite Quadrabiles, cum aliquam Quadrandam aſſumerent, quæ eorum Methodos recuſabat, ſtatim illius Quadraturam impoſſibilem eſſe crediderunt; cum exinde amplius non eſſet concludendum, qnam Methodos quibus uſi ſunt eſſe imperfectas, & ad omnes figuras non extendere. Sed cum inſtitutum meum non ſit aliorum errores detegere, aſt quid in hac materia excogitavi

B paucis

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paucis exponere; Methodum hic tradam (non ex Arithmeticis ſed Geometricis principiis deductam) quæ figurarum utriuſque generis quadraturas determinabit. Prioris generis Geometricas, poſterioris verò Algebraicas quadraturas per ſeries infinitas exhibebit. Et quia Methodus quæ ſpeciales talium figurarum quadraturas determinet, à nemine hactenus vulgata eſt, ſperamus præclarum illum Germanum (qui publice eam promiſit, & omnino in poteſtate ſua eſſe aſſeruit, in Actis Eruditorum Lipſiæ publicatis) ſuam brevi in lucem emiſſurum.

Theorema 1.

Fig. 1. SIt Curva quævis V H. (cujus axis V D, applicata H D ad V D perpendicularis) item linea V Z S talis, ut ſi à curvæ puncto libere ſumpto puta E ducatur recta E P ad Curvam, & E A Z ad axem perpendicularis, ſit recta A Z, interceptæ A P æqualis, erit ſpatium V D S $= \frac{DHq}{2}$.

Demonſtratio: Sit angulus H D O ſemirectus, & æqui ſecetur U D, indefinite punctis A, B, C per quæ ducantur, E A Z, F B Z, G C Z ad H D parallellæ, & Curvæ occurrentes in E, F, G à quibus ducantur E I Y, F K Y, G L Y, ad U D parallelæ, quin & rectæ E P, F P, G P, H P ſint Curvæ V H perpen-

perpendiculares. Eſt Triangulum HLG ſimile triangulo PDH (nam ob indefinitam ſectionem Curvula GH pro rectâ haberi poteſt) quare eſt HL. LG : : PD. DH, adeoque HL x DH = LG x PD, hoc eſt, HL x HO = DC x DS, & ſimili diſcurſu monſtrabitur, quoniam Triangulum GMF triangulo PCG aſſimilatur, fore LK x LY = CB x CZ & ſimiliter erit KI x KY = BA x BZ, itidem erit ID x IY = AV x AZ; unde conſtat triangulum HDO (quod à rectangulis iſtis HL x HO + LK x LY + KI x KY + ID x IY minime differt) æquari Spatio VDS (quod itidem à rectangulis DC x DS + CB x CZ + BA x BZ + AV x AZ minime differt) hoc eſt VDS = $\frac{DHq}{2}$. Quod erat Demonſtrandum. Nobile hoc Theorema debetur viro Celeberrimo D. Doctori *Barrow*, qui innumera habet, & ſublimia Theoremata circa linearum Curvarum proprietates; nec mihi quenquam (quorum Scripta edita ſunt) vidiſſe contigit (imo nec aliis contigiſſe puto) qui tanto judicio, & Succeſſu tanto, abſtruſiorem hanc & minus cultam Geometriæ partem, tractavit & promovit.

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Prob. I.

Data relatione inter P M (quæ diſtantiam inter Curvæ perpendicularem P C & ordinatim applicatam MC deſignat) & abſciſſam A M (quæ diſtantiam inter applicatam & verticem A deſignat) æquationem invenire Curvæ lineæ A C naturam definientem. *Fig.* 1.

UT omnes Curvas ſub unâ Regulâ generali comprehendam adnoto in quacunque linea Curva A C fore ſemper P M x M T = C M q propter angulum rectum P C T Quare multiplico ſingulos terminos P M denotantes per terminum A M (prius in diverſos numeros incognitos multiplicatum) & productum pono æquale Quadrato applicatæ C M. Ratio hujus regulæ colligi poteſt, ex Methodo inveniendi Tangentes à Clariſſimo Sluſio edita in *Actis Philoſophicis* Reg: Societatis Anglicanæ. Exemplis rem illuſtrabo.

Exemp. 1. *Fig.* 2. Detur P M $= \frac{1}{2}$ r, & vocetur AM *y*, C M *x*, a, b, c, i, &c. denotent quantitates cognitas & determinatas, item *l*, *m*, *n*, *h*. *k* &c. numeros incogitos. Jam juxta regulam multiplico $\frac{1}{2}$ r per ny, & productum $\frac{ny}{2} = x^2$. quæ eſt æquatio ad parabolam.

Exemp. 2. (*Fig.* 2.) Sit P M $= y + \frac{1}{2} r$ & quærenda ſit æquatio illam curvam determinans: procedens ſecundũ regulam

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regulam mnltiplico $\frac{1}{2}r + y$ per ny, my & productum $\frac{nry}{2} + my^2$ pono æquale quadrato ab x, nempe $\frac{nry}{2} + my^2 = x^2$ quæ eſt æquatio ab curvam quæſitam.

Exemp. 3. Eſto PM $= \frac{y^2}{a} + a$ & quæratur curva AC, in qua ſit PM $= \frac{y^2}{a} + a$. multiplico $\frac{y^2}{a} + a$ per ny, my, eritqne productum $\frac{ny^3}{a} + may = x^2$. *Fig.* 2.

Fig. 2. *Exemp.* 4. Sit PM $= \frac{y^4}{aaa} + \frac{y^3}{aa} + \frac{y^2}{a} + y$, & querenda ſit æ quatio iſtius crirvæ naturam definiens, ſecundum regulam multiplico $\frac{y^4}{a^3} + \frac{y^3}{a^2} + \frac{y^2}{a} + y$, per ny, my, ly, hy, eritque $\frac{ny^5}{a^3} + \frac{my^4}{a^2} + \frac{ly^3}{a} + hy^2 = x^2$ qua eſt æquatio quæſita.

Deniqueſit PM $= \frac{a^3}{y^2}$, multiplico $\frac{a^3}{y^2}$ per ny. erit productum $\frac{na^3}{y} = x^2$, vel $na^3 = yx^2$. *Fig.* 3
ſed quia *n*, *m*, *l*, *h*, &c. adhuc ſint incognita, modum oſtendam ea deeerminendi.

PROB. II.

Quantitates l, m, n. &c. in præcedenti Problemate uſurpatas determinare.

PEr æquationem inventam inveſtigo PM. (procedendo ſecundum vulgarem aliquam methodum inveniendi Tangentes) & ejus valorem comparo cum valore dato, nempe ſingulos hujus cum ſingulis illius terminis quæ comparatio determinabit, *l*, *m*, *n*, &c.

Ut

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Ut in exemplo primo $\frac{nry}{2} = x^2$, invenio $PM = \frac{1}{4} nr$, hunc valorem comparo cum valore dato ſc. $\frac{1}{2} r = \frac{1}{4} nr$ unde poſt reductionem $n = 2$, quare ſubſtituto hoc valore pro (n) in æquatione $\frac{nry}{2} = x^2$ erit $ry = x^2$.

Sic in exemplo ſecundo $\frac{nry}{2} + my^2 = x^2$, invenio fore $PM = \frac{nr}{4} + my$, jam comparo utroſque hos terminos cum correſpondentibus terminis valoris dati ſc. $\frac{nr}{4} = \frac{r}{2}$ unde $n = 2$, deni $my = y$ unde $m = 1$. ſi ſubſtituantur hi valores prodibit æquatio quæſita & plene determinata $ry + y^2 = x^2$.

Similiter in exemplo 3: $\frac{ny^3}{a} + may = x^2$ inveni $PM = \frac{3ny^2}{2a} + \frac{ma}{2}$, facta igitur debita comparatione ſc. $\frac{3ny^2}{2a} = \frac{y^2}{a}$ erit $n = \frac{2}{3}$, & ex $\frac{ma}{2} = a$, erit $m = 2$, & ſubſtitutis his valoribus erit $\frac{2y^3}{3a} + 2ay = x^2$.

Fig. 1. Et in exemplo 4. erit $PM = \frac{5ny^4}{2a^3} + \frac{2my^3}{a^2} + \frac{3ly^2}{2a} + hy$. & facta comparatione horum terminorum cum terminis datis erit $\frac{5ny^4}{2a^3} = \frac{y^4}{a^3}$, inde $n = \frac{2}{5}$, & ex $\frac{2my^3}{a^2} = \frac{y^3}{2a}$ erit $m = \frac{1}{2}$, ex $\frac{3ly^2}{2a} = \frac{y^2}{a}$ erit $l = \frac{2}{3}$, & ex $hy = y$ erit $h = 1$. Et ſubſtituendo hos Valores, erit equatio $\frac{2y^5}{5a^3} + \frac{y^4}{2a^2} + \frac{2y^3}{3} + y^2 = x^2$.

Denique in *Exemp.* 5. Eſt $PM = \frac{na^3}{2y^2} = \frac{a^3}{y^2}$ *Fig.* 3. unde $n = 2$, adeoque $2a^3 = yx^2$, quæ eſt ad Hyperboliformem DCE.

Hæc duo Problemata (vel potiùs duas partes unius Problematis) fuſius ſum proſecutus, eo quod à nemine

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nemine adhuc tractata ſint, ſaltem quorum ſcripta ad manus meas pervenerunt; tum maxime, quod horum ope Figurarum Quadraturas ſim determinaturus.

Prob. III.

Parabolæ Quadraturam determinare.

Fig. 4. ESTO parabola VCS cujus latus rectum ſit r. VM vocetur, y, MC, z, unde ex natura parabolæ $\sqrt{ry} = z = MC$ tum per problema primum inveniatur curva VH, talis ut ſit $PM = \sqrt{ry}$ (per PG hic & in ſequentibus intelligenda eſt Curvæ quæſitæ perpendicularis) ſed per methodum jam traditam invenio Curvam quæſitam definiri per hanc equationem $n\,r\,y^3 = x^4$ (per x deſigno applicatas GM, HD Curvæ quæſitæ) & determinando n per Prob. 2. invenies $n = \frac{16}{9}$ unde $\frac{16}{9} r\,y^3 = x^4$, ac proinde $\sqrt{\frac{4}{9} r\,y^3} = \frac{x^2}{2} = \frac{GM}{2}q =$ VMC. ut conſtat ex Theoremate jam præmiſſo.

Prob. IV.

Prob. IV.

Paraboloidis Cubicalis Quadraturam determinare.

Esto VCS parabolois Cubicalis, VD axis & latus rectum r, & VM $= y$, unde ex natura Curvæ iſtius erit $rry = z^3$ adeoque $\sqrt[3]{rry} = z$, propterea pro determinatione Quadraturæ areæ VMC invenienda eſt Curva (per Prob. 1.) VH talis ut ſit ſemper PM $= \sqrt[3]{rry}$, & procedens ſecundum regulam ibi propoſitam invenio Curvam VH definiri per hanc æquationem $nr^2y^4 = x^6$ & determinando n (per Prob. 2.) inveniens $n = \frac{27}{8}$, adeoque æquationem quæſitam eſſe $\frac{27}{8} r^2 y^4 = x^6$, unde $\frac{3}{4} \sqrt[3]{r^2 y^4} = \frac{x^2}{2} = \frac{GM}{2} q =$ VMC. Et in hunc modum Quadrantur infinitæ paraboloides quæ definiuntur per $r^3 y = z^4$; $r^4 y = z^5$, $r^5 y = z^6$, &c. *Fig.* 4.

Prob. V.

Paraboloidis Semicubicalis Quadraturam invenire.

Fig. 4. Sit VCS parabolois ſemicubicalis, cujus hæc eſt proprietas $ry^2 = z^3$, unde $\sqrt[3]{ry^2} = z^1$, invenienda igitur eſt Curva VH in

qua

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qua $PM = \sqrt[3]{r y^2} = MC$; & per problema primum invenio illam definiri per hanc æquationem $n r y^5 = x^6$; & ut determinetur n procedo in hunc modum per Prob. 2. quæro PM ex æquatione inventa $n y^5 r = x^6$; & invenio $PM = \frac{5 n r y^4}{\sqrt[3]{216 n^2 r^2 y^{10}}}$ & facta comparatione cum dato Valore erit $\frac{5 n r y^4}{\sqrt[3]{216 r^2 y^{10}}} = \sqrt[3]{r y^2}$, unde provenit $n = \frac{216}{125}$ poſt debitam reductionem, adeoque Curva VH definitur per $\frac{216 r y^5}{125} = x^6$; unde erit $\frac{3}{5} \sqrt[3]{r y^5} = \frac{x^2}{2} = \frac{GM}{2} q = VMC$. Et eodem modo Quadrari poſſunt paraboliformes infinitæ, quæ definiuntur per $r y^3 = z^4$, $r y^4 = z^5$, $r y^5 = z^6$ &c.

PROB. VI.

In Hyperbola OCN quadranda ſit Area interminata OCMVL.

Fig. 4. ESto Hyperbolæ potentia $= a^2$, unde $\frac{a^2}{y} = z$, inquirenda igitur eſt Curva VH talis ut in ea ſit ſemper $PM = \frac{a^2}{y}$ at nullam eſſe hujuſmodi Methodus jam tradita ſtatim deprehendit: nam juxta regulam multiplicanda eſt $\frac{a^2}{y}$ per $n y$, & productum

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$n a^2$ non poteſt poni æquale x^4, Quadratum determinatum nequit æquari Quadrato indeterminato, ac proinde concludendum eſt Spatium interminatum non eſſe Quadrabile: nam ſi daretur illius Quadratura, daretur etiam Curva quædam V H in qua eſſet ſemper P M = M C.

Prob. VII.

Hyberboliformis O C N cujus hæc ſit proprietas $y z^2 = a^3$ Quadraturam determinare Areæ Interminatæ OCMVL.

Fig. 5. Quoniam ex natura Curvæ $\sqrt{\frac{a^3}{y}} = z =$ MC quærenda eſt curva V H in qua ſemper ſit P M $= \sqrt{\frac{a^3}{y}}$, atque per *Prob.* 1. Invenio Curvam V H definiri per $n a^3 y = x^4$ & determinando n per *Prob.* 2. Invenies $n = 16$, adeoque $16 a^3 y = x^4$ unde Spatium interminatum O C M V L $= 2\sqrt{a y}$.

Prob. VIII.

PROB. VIII.

Sit natura Hyperboliſormis definita hac æquatione $yz^3 = a^4$, & Quadranda ſit Area interminata OCMVL.

EX natura curvæ $\sqrt[3]{\frac{a^4}{y}} = z$, & invenietur curva VH in qua $PM = \sqrt[3]{\frac{a^4}{y}}$ definiri per $27a^4y^2 = x^6$, Unde $\frac{3}{2}\sqrt[3]{a^4y^2} = \frac{x^2}{2} = \frac{GM}{2}q =$ OCMVL. Et ſic quadrantur Hyperboliformes infinitæ definitæ per $yz^4 = x^5$, $yz^5 = x^6$, $yz^6 = x^7$, &c.

PROB. IX.

In Hyperboliformi OCK cujus hæc ſit proprietas $y^2 z = a^3$ Quadranda ſit aeræ interminata OCMVL.

Fig. 6. EX hujus curvæ natura manifeſtum eſt $z = \frac{a^3}{y^2} =$ MC quare Quærenda eſt curva aliqua, ut diſtantia inter ejus perpendicularem & applicatam ſit æqualis $\frac{a^3}{y^2}$, & procedendo ſecundum uſitatam Methodum invenio Curvam quæſitam definiri per $yx^2 = na^3$ & determinando (n) per Prob. Secundum erit $n = 2$ adeoque æquatio eſt $2a^3 = yx^2$, quæ itidem eſt æquatio ad Hyberboliformem (ſed alterius naturæ) SGH; & quoniam

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illius perpendicularis G P, inter verticem & applicatam cadit, vel ſurſum tendit ideo $\frac{a^3}{y} = \frac{x^2}{2}$ = KCMD Et Aream O C M V L eſſe ex earum numero quas Geometræ vocant pluſquam infinitas, jam monuit clariſſimus Noſtras D. *David Gregorius* in pulcherrimo ſuo Tractatu, de Dimenſione Figurarum.

Prob. X.

Sit A C D curva talis ut ducta ut cunque M C ad A D normali, ſit poteſtas quævis ipſius A D ad ſimilem poteſtatem partis A M, ut poteſtas quævis partis D M ad ſimilem poteſtatem applicatæ M C, & determinanda ſit Quadratura Areæ A M C.

Fig. 7. ESTO AD = b, & exponens illius poteſtatis 2, A M vocetur y unde etiam exponens illius poteſtatis eſt, 2 eſto præterea exponens poteſtatis lineæ D M ſeu $b - y$, (1) adeoque exponens applicatæ M C ſeu z eſt 1, tum ex naturæ lineæ curva. $b^2 \, y^2 :: b - y . z$ unde $z = \frac{by^2 - y^3}{b^2}$, Quæratur ergo curva AH in qua ſit P M = $\frac{by^2 - y^3}{b^2}$, inveniaturque per Prob. 1. & 2. illam definiri per $\frac{2}{3} b y^3 - \frac{1}{2} y^4 = b^2 x^2$, *unde* $\frac{y^3}{3b} - \frac{y^4}{4bb} = \frac{x^2}{2}$ = A MC. Atque hæc eadem eſt curva de qua loquitur D. *Carteſius* in *tom.* 3. Epiſt. pag. 219. quam præferendam putat (ob conſtructionis facilitatem) curvæ quam Galli vocant *la Galande*.

Prob. XI.

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Prob. XI.

Determinanda ſit Quadratura Areæ AMC, & definiatur natura Curvæ per $y^5 + ay^4 + a^2y^3 + a^3y^2 + a = a^4 z$. Fig. 4.

Quærenda eſt Curva VH, talis ut in ea ſit ſemper $PM = MC = z = \frac{y^5}{a^4} + \frac{y^4}{a^3} + \frac{y^3}{a^2} + \frac{y^2}{a} + a$. Et per Prob. 1. definietur per $ny^6 + may^5 + la^2y^4 + ka^3y^3 + ha^5y = a^4x^2$ & determinando n, m, l, k, h (per *Prob.* 2.) erit $n = \frac{1}{3}$ $m = \frac{2}{5}$, $l = \frac{1}{2}$, $k = \frac{2}{3}$, $h = 2$; adeoque æquatio quæſita eſt $\frac{1}{3}y^6 + \frac{2}{5}ay^5 + \frac{1}{2}a^2y^4 + \frac{2}{3}a^3y^3 + 2a^5y = a^4x^2$, adeoque $\frac{y^6}{6a^4} + \frac{y^5}{5a^3} + \frac{y^4}{4a^2} + \frac{y^3}{3a} + ay = \frac{x^2}{2} = \frac{OMq}{2} = AMC$.

Atque hactenus ſolas illas figuras tractavi quæ ſunt indefinitè Quadrabiles, & quantillo labore earum Quadraturæ, per hanc Methodum determinentur, aliis judicandum relinquo: Ad illas jam progredior quæ hujuſmodi Quadraturam reſpuunt: & expreſſe moneo me Quadraturas quas hic exhibiturus ſum per ſeries infinitas, non pro Geometricis ſed Algebraicis vel Arithmeticis habere.

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Prob. XII.

Circuli Quadraturam determinare.

A Circulo initium faciam, qui omnium linearum curvarum ſimpliciſſima eſt, ſi curvæ ſimplicitas non ex æquationis, ſed deſcriptionis (ut re verâ debet) ſimplicitate æſtimetur.

Fig. 8. Sit itaque Circuli Quadrans ASD in quo AM vocetur y, & ordinata MC z, & radius AL $= r$, tum ex Circuli natura erit $z^2 = r^2 - y^2$, ac proinde $z = \sqrt{r^2 - y^2}$ hunc valorem reſolvo in ſeriem ſecundum Methodum celeberrimi D. *Iſaaci Newtoni*, & invenio $z = r - \frac{y^2}{2r} - \frac{y^4}{8r^3} - \frac{y^6}{16r^5}$ ---- &c. quærenda igitur eſt Curva AGH in qua PM $= r - \frac{y^2}{2r} - \frac{y^4}{8r^3} - \frac{y^6}{16r^5}$ ---- &c. & invenietur per Prob. primum Curvam quæſitam definiri per hanc æquationem.

$$nry - \frac{my^3}{2r} - \frac{ly^5}{8r^3} - \frac{ky^7}{16r^5} = x^2;$$ & determinando Quantitates, n, m, l, k, per Prob. ſecundum invenietur $n = 2$, $m = \frac{2}{3}$, $l = \frac{2}{5}$, $k = \frac{2}{7}$, & ſubſtituendo hos valores, æquatio erit $2ry - \frac{y^3}{3r} - \frac{y^5}{20r^3} - \frac{y^7}{56r^5} = x^2$. Unde $ry - \frac{y^3}{6r} - \frac{y^5}{40r^3} - \frac{y^7}{112r^5} = \frac{x^2}{2}$

= GMq

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$= \frac{GMq}{2} =$ AMCS. Vel ſi quæreretur Quadratura totius Quadrantis, erit ASD $= r^2 - \frac{1}{6}rr - \frac{1}{40}rr - \frac{1}{112}rr$ ---- &c. Unde $4rr - \frac{2}{3}rr - \frac{1}{8}r^2 - \frac{1}{28}r^2 =$ toto Circulo. Et ſi hæc ſeries per numeros exprimatur, ponendo $r = \frac{1}{2}$ erit area Circuli $= 1 - \frac{1}{4} - \frac{1}{32} - \frac{1}{112}$ ---- &c. in infinitum Notatu dignum arbitror hinc elici poſſe dimenſionem Zonæ Circularis, quam à celeberrimo Geometra D. *Iſaaco Newtono* inventam refert clariſſ. *David Gregorius* in memorato tractatu. Eſto ABCD Zona cujus latitud. VL $= y$, & Circuli radius $= r$, per præcedentem Quadraturam ---- VBCL $= ry - \frac{y^3}{6r} - \frac{y^5}{40r^3} - \frac{y^7}{112r^5}$ -- Adeoque -- 2 VBCL $=$ ABCD $= 2ry - \frac{y^3}{3r} - \frac{y^5}{20r^3} - \frac{y^7}{56r^5}$.

Fig. 9.

Prob. XIII.

Hyperbolæ Quadraturam determinare.

Fig. 10.

SIT LSC Hyperbola cujus Aſymptoti VD, VP, & in qua VE $=$ EL $= a$, atque VA $= c$, vocetur abſciſſa AM y, & ordinatim applicata z, ſed ex natura Hyperbolæ VE $\times$ EL

$=$ VM

$= VM \times MC$, id eſt $a^2 = yz + cz$; adeoque eſt $z = \frac{a^2}{c+y}$, & facta diviſione, ſecundum jam receptam Methodum, erit $z = \frac{a^2}{c} - \frac{a^2 y}{c^2} + \frac{a^2 y^2}{c^3}$ --- &c. Quærenda igitur eſt Curva AGH, in qua ſit ------ $PM = \frac{a^2}{c} - \frac{a^2 y}{c^2} + \frac{a^2 y^2}{c^3}$ ----- &c. & per Prob. primum invenietur illam definiri hac æquatione $\frac{n a^2 y}{c} - \frac{m a^2 y^2}{c^2} + \frac{l a^2 y^3}{c^3} = x^2$, & determinando, n, m, l, per Prob. 2. erit $n = 2$, $m = 1$, $l = \frac{2}{3}$, ac proinde æquatio quæſita eſt $\frac{2 a^2 y}{c} - \frac{a^2 y^2}{c^2} + \frac{2 a^2 y^3}{3 c^3} = x^2$ unde $\frac{a^2 y}{c} - \frac{a^2 y^2}{2 c^2} + \frac{a^2 y^3}{3 c^3} = ASCM = \frac{x^2}{2} = \frac{GMq}{2}$. Hæc eadem eſt Hyperbolæ Quadratura quam exhibuit celebris vir *Nicolaus Mercator* in ſua Logarithmo-technia, quamvis methodo uſus ſum ab illius plane diverſâ.

Conſiderando aliam Hyperbolæ proprietatem; aliam etiam illius Quadraturam inveniemus. Sit ergo in *Fig.* 11. appoſito ſchemate SCL Hyperbola æquilatera cujus centrum A & latus tranſverſum RS, ponatur $AM = y$, $= KC$, $MC = z$, $AR = AS$

$= AS = r$, unde ex natura Hyperbolæ $rr + yy = zz$, adeoque $z = \sqrt{rr+yy}$, extrahendo radicem Quadraticam ex $\sqrt{rr+yy}$ erit $z = r + \frac{y^2}{2r} - \frac{y^4}{8r^3} + \frac{y^6}{16r^5}$ ---- &c. Quærenda ergo eſt Curva AH in qua ſit $PM = z = r + \frac{y^2}{2r} - \frac{y^4}{8r^3} + \frac{y^6}{16r^5}$ ---- &c. & procedendo per Prob. primum invenietur illam definiri hac æquatione $nry + \frac{my^3}{2r} - \frac{ly^5}{8r^3} + \frac{ky^7}{16r^5} = x^2$, & determinando, n, m, l, k, per Prob. ſecundum, erit $n = 2$, $m = \frac{2}{3}$, $l = \frac{2}{5}$, $k = \frac{2}{7}$, ſubſtitutis his valoribus erit æquatio ad Curvam quæſitam plene determinata $2ry + \frac{y^3}{3r} - \frac{y^5}{20r^3} + \frac{y^7}{56r^5} = x^2$, adeoque erit ---- $ry + \frac{y^3}{6r} - \frac{y^5}{40r^3} + \frac{y^7}{112r^5} = \frac{x^2}{2} = \frac{GMq}{2} = ASCM$.

Ex hac Hyperbolæ Quadratura facile eſt Zonæ Hyperbolicæ Quadraturam determinare. Sint EDA, GCB Hyperbolæ oppoſitæ, quarum centrum K & vertices A, B, Zona ABCD cujus latitudo $KL = y$, ſemiaxis tranſverſus AK, vel $KB = r$, unde per præcedentem Quadraturam *Fig. 12.*

KLCB

$KLCB = ry + \frac{y^3}{6r} - \frac{y^5}{40r^3} + \frac{y^7}{112r^5}$ ac proinde erit $ABCD = 2ry + \frac{y^3}{3r} - \frac{y^5}{20r^3} + \frac{y^7}{56r^5}$ ---- &c.

Prob. XIV.

Ellipſeos Quadraturam determinare.

Fig. 8. IN ſemi-ellipſi LSCD ſit ſemiaxis tranſverſus AS = b & ſemiaxis conjugatus AL = a, & ponatur abſciſſa AM = y, ordinatim applicata MC = z, unde ex natura Ellipſeos $z = \frac{b}{a}\sqrt{a^2 - y^2}$, ut ergo determinetur Area AMCS, primo reſolvenda eſt $\frac{b}{a}\sqrt{a^2 - y^2}$, in ſeriem extrahendo radicem ex $\sqrt{a^2 - y^2}$, unde invenietur $z = b - \frac{by^2}{2a^2} - \frac{by^4}{8a^4} - \frac{by^6}{16a^6}$ ----- &c. Quærenda igitur eſt Curva aliqua AH in qua ſemper PM $= b - \frac{by^2}{2a^2} - \frac{by^4}{8a^4} - \frac{by^6}{16a^6}$ ---- &c. & invenietur per Prob. 1. illam definiri hac æquatione $nby - \frac{mby^3}{2a^2} - \frac{lby^5}{8a^4} - \frac{kby^7}{16a^6} = x^2$ & determinando quan-

titates,

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titates, n, m, l, k per Prob. 2, erit $n = 2$, $m = \frac{2}{3}$ $l = \frac{2}{5}$, $k = \frac{2}{7}$, adeoque $2by - \frac{by^3}{3a^2} - \frac{by^5}{20a^4} - \frac{by^7}{56a^6} = x^2$ unde $by - \frac{by^3}{6a^2} - \frac{by^5}{40a^4} - \frac{by^7}{112a^6} = \frac{x^2}{2} =$ $\frac{GMq}{2} = AMSC$.

Inde facile eruitur Zonæ Ellipticæ dimenſio, ut ſi latitudo Zonæ ſit $AM = y$, cæteris poſitis ut prius erit Zona $2\,AMCS = 2by - \frac{by^3}{3a^2} - \frac{by^5}{20a^4} -$ $\frac{by^7}{56a^6} - -$ &c.

Prob. XV.

Sit AD ($= d$) poſitione & magnitudine data, & Curva SCD talis ut ea ducta utcunque recta MC ($= z$) ad AD perpendicularis ſit $d^3 = z^3 + y^3$, & determinanda ſit Areæ AMCS Quadratura. Fig. 13.

Quoniam ex natura Curvæ $z = \sqrt[3]{d^3 - y^3}$ extrahenda eſt radix Cubica ex $d^3 - y^3$, & invenietur fore $z = d - \frac{y^3}{3d^2} - \frac{y^6}{9d^5} -$ &c.

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Quærenda eſt linea Curva AGH in qua $PM = d - \frac{y^3}{3d^2} - \frac{y^6}{9d^5} - -$ &c. & definietur Curva quæſita AH hac æquatione $n\,dy - \frac{my^4}{3d^2} - \frac{ly^7}{9d^5} = x^2$ & determinando n, m, l, per problema ſecundum erit $n = 2$, $m = \frac{1}{2}$, $l = \frac{2}{7}$ adeoque $2\,dy - \frac{y^4}{6d^2} - \frac{2y^7}{63d^5} = x^2$, unde etiam $dy - \frac{y^4}{12d^2} - \frac{y^7}{63d^5} = \frac{x^2}{2} = \frac{GMq}{2} =$ AMCS. Et ſic Quadrantur Cycliformes infinitæ quæ definiuntur per $d^4 - y^4 = z^4$, $d^5 - y^5 = z^5 - -$ &c.

PROB. XVI.

Eſto AD (= d) linea recta poſitione & magnitudine data, & SCD linea Curva talis ut ducta utcunque MC (= z) ad AD normali ſit Cubus ex AD cum Cubo ex AM (= y) æquales Cubo ex MC ſc. $d^3 + y^3 = z^3$ *& determinanda ſit Quadratum Areæ AMC.*

Fig. 13.

QUoniam $z = \sqrt{(3)\,d^3 + y^3}$ reſolvenda eſt $\sqrt{(3)\,d^3 + y^3}$ in ſeriem radicem cubicam extrahendo, & invenietur $z = d + \frac{y^3}{3d^2} - \frac{y^6}{9d^5} - - -$ &c. Quærenda

eſt

eſt Curva AH in qua ſit ſemper PM $= d + \frac{y^3}{3d^2} - \frac{y^6}{9d^5} - -$ &c. & per Prob. 1. & 2. erit æquatio ad Curvam quæſitam ---- $2dy + \frac{y^4}{6d^2} - \frac{2y^7}{63d^5} = x^2$ unde $dy + \frac{y^4}{12d^2} - \frac{y^7}{63d^5} = \frac{x^2}{2} =$ AMCS. Et ſic Quadrari poſſunt Hyperboliformes infinitæ quæ definiuntur per $d^4 + y^4 = z^4$, $d^5 + y^5 = z^5$, &c.

Prob. XVII.

Sit $AD = a$ $AS = b$, & ſit Curva SCD talis ut ducta quavis MC ($= z$) ad AD perpendiculari ſit $z^3 . a^3 - y^3 :: b^3 . a^3$, & determinanda ſit Area AMCS. Fig. 13.

Quoniam ex natura Curvæ $z = \frac{b}{a} \sqrt{(3)\, a^3 - y^3}$ extrahenda eſt radix ex $a^3 - y^3$, invenieturque $z = b - \frac{by^3}{3a^3} - \frac{by^6}{9a^6} - -$ &c. Quærenda eſt Curva AH in qua PM $=$ MC $= b - \frac{by^3}{3a^3} - \frac{by^6}{9a^6}$, &c. & per Prob. 1 & 2. invenietur Curvam quæſitam definiri hac æquatione $2by - \frac{by^4}{6a^3} - \frac{2by^7}{63a^6} = x^2$, ac propterea $by - \frac{by^4}{12a^3} - \frac{by^7}{63a^6} = \frac{x^2}{2} = \frac{GMq}{2} =$ AMCS. Et ſic quadrantur Ellipſiformes infinitæ, quæ definiuntur æquationibus $\frac{b}{a} \sqrt{(4)\, a^4 - y^4} = z$, $\frac{b}{a} \sqrt{(5)\, a^5 - y^5} = z$, &c.

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PROB. XVIII.

Eſto AD (= d) linea recta poſitione & magnitudine data & SCD Curva talis ut ducta utcunque MC ad AD normali ſit $d^2 z + y^2 z = r^3$, & Quadranda ſit Area AMCS.

Fig. 13.

QUoniam $z = \frac{r^3}{d^2 + y^2}$; fiat diviſio, & invenietur $z = \frac{r^3}{d^2} - \frac{r^3 y^2}{d^4} + \frac{r^3 y^4}{d^6}$; eritque $\frac{2r^3 y}{d^2} - \frac{2r^3 y^3}{3d^4} + \frac{2r^3 y^5}{5d^6} = x^2$ æquatio ad Curvam AH in qua $PM = \frac{r^3}{d^2} - \frac{r^3 y^2}{d^4} + \frac{r^3 y^4}{d^6}$ unde $\frac{r^3 y}{d^2} - \frac{r^3 y^3}{3d^4} + \frac{r^3 y^5}{5d^6} = \frac{x^2}{2} = \frac{GMq}{2} =$ AMCS.

PROB. XIX.

Cujuſvis Figuræ Quadraturas infinitas invenire.

SInt duæ quælibet Curvæ ACE, BRF, & à quovis puncto C in curva ACE ducatur tangens CT, & CP ad EP, & CR ad AB parallela, ſi fiat TP . PC :: DR . PX, erit ABZY = BEF, APXY = DBR. Eximium hoc Theorema debetur etiam viro celeberrimo D. Doctori *Barrow*.

Quærenda ſint, exempli gratia, Quadratura infinitæ Paraboloidis Cubicalis BRF. Aſſumatur pro

arbitrio

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arbitrio quælibet Curva ACE puta parabola communis cujus parameter ſit r (quod idem ſit cum parametro paraboloidis) & ponatur $AP = y$, $PX = z$, eritque $TP = 2y$, $PC = \sqrt{ry}$, & ex natura paraboloidis $DR = \sqrt{(6)}\, r^5 y$ adeoque Analogia erit $2y . \sqrt{ry} :: \sqrt{(6)}\, r^5 y . z$ unde $z = \sqrt{(6)} \frac{r^8}{64y^2}$ quæ eſt æquatio ad Curvam YXZ, cujus Quadratura invenietur per methodum jam traditam ſc. $\frac{1}{4}\sqrt{(6)}\, r^9 y^4 = APYZ = DBR$. Quæ eſt Quadratura Paraboloidis diverſa ab illa quam dedi in Prob. 4. Et eodem modo inveniri poteſt alia atque alia Quadratura, aſſumendo aliam atque aliam Curvam ACE. Et ſic tractari poſſunt omnes aliæ Curvæ, quemadmodum paraboſidem hic tractavi.

Exhinc etiam manifeſtum eſt Figuras deprimi poſſe ad ſimpliciores & Quadratu faciliores, nam in figura ABZY Curva YXZ definita hac æquatione $z^6 = \frac{r^8}{64y^2}$ magis eſt compoſita quam Curva BRF. Adeoque non parum Geometriam promoveret, qui methodum daret Figuras ad ſimpliciſſimas reducendi.

Prob. XX.

Prob. XX.

Curvam invenire cujus Area per datam quamlibet æquationem deſignetur.

DEſignetur Area hac æquatione $\sqrt{r^3 y}$ = VMC (concipiendo VCS eſſe Curvam quæſitam.) Tum ex oſtenſis patet $\sqrt{r^3 y} = \frac{x^2}{2}$ eſſe æquationem ad Curvam aliam VGH in qua PM = MC (quæ eſt ordinata Curvæ quæſitæ) inveſtigetur ergo valor lineæ PM, & invenietur PM = $\sqrt{\frac{r^3}{4y}} = z$ ſeu $4z^2y = r^3$, quæ eſt æquatio ad Curvam quæſitam VCS cujus area = $\sqrt{r^3 y}$. Notandum quod hic (ut prius) y denotat abſciſſas VM, z ordinatas MC, & x ordinatas GM.

Prob. XXI.

Curvas infinitas invenire quarum Areæ per unam datam æquationem deſignentur.

SOlutio hujus problematis à duobus præcedentibus pendet; inveniatur una Curva cujus Area per datam æquationem exprimatur (per problema 20) & ſic infinitæ inveniri poſſunt per Prob. 19.

Prob. XXII.

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PROB. XXII.

Data qualibet Curva AHD Curvam aliam AFB invenire cujus area AGF æquetur rectangulo contento ſub ordinata GH & abſciſſa AG Curvæ datæ. Fig. 20.

IN Curva AHD ſit $AG = y$, $GH = x$, & exprimatur illius natura hac æquatione $2ay - yy = x^2$, unde $\sqrt{2ay - y^2} = x$ adeoque $\sqrt{2ay^3 - y^4} = xy = AH$. Habetur ergo Area figuræ AGF, unde facile Curva AFB definitur per problema 20. ſc.

$$z^2 = \frac{9a^2y - 12ay^2 + 4y^3}{2a - y}.$$

PROB. XXIII.

Data qualibet Curva AHD, aliam Curvam AFB invenire cujus area AGF æquatur rectangulo contento ſub ordinata GH Curvæ AHD, & conſtanti aliqua data recta (a) Fig. 20.

DEfiniatur Curva AHD ut prius $\sqrt{2ay - yy} = x$ unde $\sqrt{2a^3y - a^2y^2} = ax = AGF$, habetur ergo natura Curvæ AFB per Prob. 20. ſc.

E $z^2 =$

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$z^2 = \frac{a^4 - 2r^2y^2 + a^2y^2}{2ay - y^2}$ hic & in precedenti z denotat ordinatas Curvæ quæſitæ A F B.

Et quidem aliis modis infinitis (præter duos jam traditos) inveniri poteſt curva cujus area, ope alterius curvæ datæ ſit quadrabilis, per Prob. 20. Quod fieri poſſe aſſeruit jam laudatus Germanus, ſed quo modo faciendum ſit nequaquam oſtendit.

Alia ſolutio problematis præcedentis.

Fig. 21. SIT Curva data A C B, C T tangens in puncto quolibet C, ordinata C F; fiatq; T F. F C : : a. F Z, orietur hinc Curva A Z Z talis ut a + F C = A F Z; ut demonſtratum eſt ab illuſtriſſimo D. Doctori *Barrow*.

Nec quidquam jam deeſt ut Methodus quam tradidi Figurarum Quadraturas determinandi, ad omnes figuras extendatur (exceptis iis quæ a Curvis tranſcendentibus terminantur, quas nulla hactenus vulgata Methodus comprehendit) niſi ut difficultates duas amoveam; quæ in quibuſdam caſibus contingere poſſunt; quarum prior accidit cum figuram aliquam Quadrando neceſſe ſit & Radicem ex æquatione effecta (& ſupra Quadraticas aſcendente) extrahere, in quo caſu unicum remedium mihi cognitum eſt radicem iſtius æquationis in ſeriem infinitam (juxta Methodum clariſſimi viri D. Iſaaci Newtoni Geometræ non minus

quam

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quam Analyſtæ præſtantiſſimi) reſolvere, quam prælo commiſſam eſſe à clariſſ. Walliſio audimus, quamque inſignis ipſe D. Newtonus mihi in Manuſcriptis pro ſummâ ſua humanitate communicavit: Nam Methodus Generalis æquationum radices Analyticè determinandi (in Actis Eruditorum Lipſiæ publicatis A° 1683, Menſe Maio a præclaro illo Germano edita) huic negotio parum vel nihil inſervit, ut de inſuperabili in eâ calculi moleſtia nihil dicam. Sed nihilominus inventum eſt inter præcipua Artis Analyticæ merito numerandum.

Secunda difficultas eſt cum valor ordinatim applicatæ conſtat terminis aſymmetris, nam res eſſet immenſi laboris æquationem ab aſymmetria liberare, ſi plures ſint quàm quatuor termini ſignis radicalibus affecti, ut ſatis norint Analyſeos periti. Sed huic difficultati remedium optimum ſuppeditavit inſignis Geometra G. G. Leebnitius in nova ſua Methodo Tangentes inveniendi in Actis Eruditorum Anni ſuperioris publicata. Ibi enim præclarus vir viam expeditam oſtendit, Tangentes inveniendi, quamvis æquatio curvæ naturam exprimens terminis irrationalibus quam maximè ſit implicita, non ablatis irrationalibus. Quomodo iſta methodus ad præſens negotium ſit applicanda exemplo oſtendam.

Eſto V C S circuli Quadrans cujus Diameter ſit (*r*) & V M vocetur *y*, item ordinata M C *Fig. 4.*

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z, tum ex natura circuli $z = \sqrt{ry - yy}$ & reſolvendo $ry - y^2$ in ſeriem per extractionem radicis, invenietur. $z = \sqrt{ry} - \sqrt{\frac{y^3}{4r}}\sqrt{\frac{y^5}{16r^3}}$.....&c. Ut determinetur Quadratura areæ V M C, invenienda eſt curva V H, in quâ $PM = \sqrt{ry} - \sqrt{\frac{y^3}{4r}} - \frac{y^5}{16r^3}$, eritq; per Prob. 1. æquatio ad curvam quæſitam V H $\sqrt{nry^3} - \sqrt{\frac{my^5}{4r}} - \sqrt{\frac{ly^7}{16r^3}} = x^2$; & auferendo quantitates fractas (quod tamen abſolute neceſſe non eſt, ſed hic fit ob majorem facilitatem) multiplicando per $\sqrt{16r^3}$: erit $\sqrt{16nr^4y^3} - \sqrt{4mr^2y^5} - \sqrt{ly^7} = x^2\sqrt{16r^3}$; & determinando n, m, l, (per Prob. 2.) quæ ſola eſt difficultas ſic procedo: compendii cauſa pono $p = 16nr^4y^3$ $q = 4mr^2y^5$, $s = ly^7$; eritq; $\sqrt{p} - \sqrt{q} - \sqrt{s} = x^2\sqrt{16r^3}$; ſed per calculum ibi explicatum invenietur $\sqrt{p} = \frac{dp}{\sqrt{4p}}$, $\sqrt{q} = \frac{dq}{\sqrt{4q}}$, $\sqrt{s} = \frac{sds}{\sqrt{4s}}$, atque $x^2\sqrt{16r^3} = 2x\sqrt{16r^3}\,dx$, & ſubſtitutis his valoribus erit $\frac{dp}{\sqrt{4p}} - \frac{dq}{\sqrt{4q}} - \frac{ds}{\sqrt{4s}} = 2x\sqrt{16r^3}\,dx$; ſed per eundem calculum erit $dp = 48nr^4y^2\,dy$, $dq = 20mr^2y^4\,dy$ & deniq; $ds = 7ly^6\,dy$, & ſubſtituendo hos valores cum valoribus quantitatum $\sqrt{4p}$, $\sqrt{4q}$, $\sqrt{4s}$, æquatio erit $\frac{48nr^4y^2dy}{\sqrt{64nr^4y^3}} - \frac{20mr^2y^4dy}{\sqrt{16mr^2y^5}} - \frac{7ly6.dy}{\sqrt{4ly7}} = 2x\sqrt{16r^3}\,dx$.———

Quam clariſſimus Author æquationem differentialem appellat: & hæc æquatio in Analogiam reſoluta dat. $dy.dx :: x\sqrt{64r^3}.\ \frac{48nr^4y^2}{\sqrt{64nr^4y^3}} \frac{20mr^2y^4}{\sqrt{16mr^2y^5}} \frac{7ly6}{\sqrt{4ly7}} ::$ x. PM. ut ex eodem calculo eſt manifeſtum, adeoq; erit

PM=

$PM = \frac{48 n r^4 y^2}{\sqrt{4096 n r^7 y^3}} - \frac{20 m r^2 y^4}{\sqrt{1024 m r^5 y^5}} - \frac{7 l y^6}{\sqrt{256 l r^3 y^7}} = \&c.$

Et facta comparatione horum terminorum cum terminis prioribus P M denotantibus, juxta cognitas comparationis leges ſc. $\frac{48 n r^4 y^2}{\sqrt{4096 r^7 y^3}} = \sqrt{ry}$, inde $n = \frac{16}{9}$ ſimiliter $m = \frac{16}{25}$; & $l = \frac{16}{49}$, quibus ſubſtitutis erit $\sqrt{\frac{16 ry}{9}} - \sqrt{\frac{16 y^5}{100 r}} - \sqrt{\frac{y^7}{496 r^3}} = x^2 = GMq$, adeoque $\sqrt{\frac{4 ry^3}{9}} - \sqrt{\frac{16 y^5}{400 r}} - \sqrt{\frac{y^7}{196 r^3}} = \frac{x^2}{2} = \frac{CMq}{2} =$ V M C. Adeoq; etiam hoc modo habetur circuli Quadratura. Et ſimilem diſcurſum in aliis adhibere non erit difficile, cuivis in ſingulari hoc calculi genere verſato, ita ut ſuperfluum duxi præſtantiſſimæ hujus Methodi uſum pluribus exemplis illuſtrare. Unum tamen eſt quod hic obiter notandum puto, poſſe ex hac Tangentium methodo breviter demonſtrari veritatem Regulæ quam dedi pro ſolutione problematis primi.

Namq; $dy . dx$: : T M. M C (ut ex iſtá methodo eſt manifeſtum) ſed T M. M C : : M C. P M. ob angulum rectum T M P. Ergo $dy . dx$: : M C. P M. (vel poſito x pro M C) erit $dy . dx$: : x. P M. Unde P M $x dy = x dx$, & ſubſtituendo y & x pro earum differentiis dy, dx erit P M $x . y = x^2$ Quod demonſtrandum erat.

Jamq; concludo, ſi nulla ſit Curva in qua diſtantia inter illius perpendicularem & ordinatam ſit æqualis

lis correſpondenti ordinatæ in Curva Figuram (cum recta vel rectis) comprehendentem, illam Figuram non eſſe indefinite Quadrabilem; nam ſi daretur illius Quadratura indefinita, daretur etiam hujuſmodi Curva ut patet ex *Prob.* 20. Et nullam eſſe talem Curvam pro Circulo & Hyperbola, facile poſſum demonſtrare, ſed demonſtrationem ob nimiam prolixitatem hic omitto.

De Linearum Curvarum Rectificatione.

Quiſnam fuerit qui primò Curvæ rectam æqualem invenit diu multumq; Anglos inter & Batavos diſputatum fuit; & qui plenius de ea re ſibi ſatisfieri volunt, totam diſputationem videre poſſunt, in eximio libello de Cycloide a Clariſſ. Walliſio Edito *pag.* 91, 92, 93, *&c.* itemq; in Horologio Oſcillatorio illuſtriſſimi Hugenii *pag.* 72, 73, & deniq; in Epiſtola Walliſii in Actis Philoſophicis Reg. Societ. publicata Num. 98. res enim tanti non eſt ut ulteriori diſquiſitione digna videatur, mihi præſertim qui nec Anglus ſum nec Batavus. Ea tamen, quæ, re bene perpenſa, utrinq; manifeſta videntur, breviter annotabo: 1. Quod Guliel. Nelius Equitis Angli Filius omnium primus rectam Curvæ æqualem invenerit.

2. Quod non datam Curvam rectificaverit ſed Curvam rectificationis capacem exhibuerit. 3. Quod dig-

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niſſimus & Geometra periti ſſimus D. Chriſtoph. Wren primo oblatæ Curvæ (ſc. Cycloidi) rectam æqualem determinaverit. 4. Quod Heuratius primo oſtenderit quamlibet datam Curvam rectificare, ſuppoſitis Figurarum Quadraturis. Et in eo Heuratii Methodi non parum eſt conſpicua quod ſtatim indicat quænam illa Figura ſit cujus Quadratura Curvam datam rectificaret; Adeoq; cum jam Methodum generalem præmiſi Figurarum Quadraturas determinandi; facile erit Curvam aliquam in rectam tranſmutare; Et recta illa vel per æquationem finitam (cum nempe Figura eſt indefinite Quadrabilis) vel per ſeriem infinitam exprimetur. Henratius enim tali methodo deſtitutus, non potuit methodum ſuam Curvas rectificandi, ad omnes illas Curvas extendere, Quarum rectificationes a figuris indefinitè Quadrabilibus dependent; multoq; minus cum à figura ſpecialis tantum Quadraturæ capaci dependerent.

THEOREMA 2.

Fig. 14. SINT duæ lineæ Curvæ A C E, G I L, & recta A F ejus naturæ ut (ducta ex puncto M libere ſumpto perpendiculari *MI* ſecante Curvas in C & I, uti & C P perpendiculari ad Curvam A C E) ſit .... *MC*. C P : : R. *MI* (R hic eſt quælibet linea recta data vel aſſumpta) erit AGILEF=

Rx

R x ACE. Demonſtratio hujus Theorematis habetur in Epiſtola Heuratii ad Schotenium.

Prob. I.

Determinare Longitudinem Parabolæ. ACE.

Fig. 14. SIT parabolæ vertex *A*, ipſius axis *AG*, & parameter (*a*) *AM* vocetur *x* & *MC* vocetur *y*; unde ex natura parabolæ $x^2 = ay$; per methodum aliquam vulgarem Tangentes inveniendi, conſtabit fore $PM = \frac{2x^3}{a^2}$, adeoq; $PMq = \frac{4x^6}{a^4}$, unde $PC = \sqrt{\frac{4x^6}{a^2} + \frac{x^4}{a}}$ jam quia CM. CP :: *a*. MI; vel in terminis Analytricis $\frac{x^2}{a}$. $\sqrt{\frac{4x^6}{a^4} + \frac{x^4}{a^2}}$ :: *a*. *z* (poſito nimirum $MI = z$) unde $z = \sqrt{a^2 + 4x^2}$, quæ eſt æquatio ad Hyperbolam; adeoq; pro determinatione longitudinis lineæ parabolicæ ACE, Quadranda eſt Area Hyperbolica AGILEF (ut in *Prob.* 13.) eritque,

$AGILF = ax + \frac{2x^3}{3a} - \frac{2x^5}{5a^3} + \frac{4x^7}{7a^5}$ ——— &c.

Unde $ax ACE = ax + \frac{2x^3}{3a} - \frac{2x^5}{5a^3} + \frac{4x^7}{7a^5}$ — &c. per Theor. 2.

Adeoq; $ACE = x + \frac{2x^3}{3a^2} - \frac{2x^5}{5a^4} + \frac{4x^7}{7a^6}$ ——— &c.

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PROB. II.

Circuli Peripheriæ rectam æqualem exhibere.

SIT ACF circuli Quadrans cujus radius ſit d. & vocetur PM y, MCx; & MI z, ſitq; GIL talis curva ut ducta utcunq; normali CMI ad rectam PF ſit MC. PC : : d (recta libere ſumpta). MI. id eſt $\sqrt{dd-yy}$. d : : d. z. unde $z = \frac{dd}{\sqrt{dd-yy}}$ quæ eſt æquatio ad curvam GIL; adeoq; PM16 $= dy + \frac{y^3}{6d} + \frac{3y^5}{40d^3} + \frac{5y^7}{112d^5}$ — — &c. Sed per Theor. 2. eſt dx AC $= dy + \frac{y^3}{6d} + \frac{3y^5}{40d^3} + \frac{5y^7}{112d^5}$ — &c. Ergo AC $= y + \frac{y^3}{6d^2} + \frac{3y^5}{40d^4} + \frac{5y^7}{112d^6}$ — — &c. Fig. 15.

PROB. III.

Hyperbolæ rectam æqualem exhibere.

SIT ACE Hyperbola æquilatera cujus ſemiaxis BA $= a$ & centrum B; & BM vocetur y, ACx, unde ex natura Hyperbolæ $a^2 + y^2 = x^2$; ponatur PC Hyperbolæ in C perpendicularis; Fig. 16.

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invenietur $PM = y$ adeoq; $PC = \sqrt{a^2+2y^2}$: ſi fiat MC. CP :: a. M. id eſt, $\sqrt{a^2+y^2}$. $\sqrt{a^2+2y^2}$ :: a. z; erit $z = \frac{\sqrt{a^4+2a^2y^2}}{\sqrt{a^2+y^2}}$ quæ eſt æquatio ad Curvam GIL.

Sed $\sqrt{a^4+2a^2y^2} = a^2 + y^2 - \frac{y^4}{2a^2}$ ——— &c.

Et $\sqrt{a^2+y^2} = a + \frac{y^2}{2a} - \frac{y^4}{8a^3}$ ——— &c.

Ideoq; $\frac{\sqrt{a^4+2a^2y^2}}{\sqrt{a^2+y^2}} = a + \frac{y^2}{2a} - \frac{5y^4}{8a^3}$ ——— &c.

Unde $BELG = ay + \frac{y^3}{6a} + \frac{5y^5}{40a^3}$ ——— &c.

Atq; $ACE = y + \frac{y^3}{6a^2} - \frac{5y^5}{40a^4}$ ——— &c.

De Curvarum ſuperficierum dimenſione.

QUemadmodum linearum Curvarum longitudines, ſic etiam ſuperficierum, quæ ab illarum rotatione generantur, dimenſio ex quarundam Figurarum Quadraturis dependet, ut ex ſequenti Theoremate conſtat.

THEOREMA 3.

Fig. 17. SIT MP Curvæ AMB perpendicularis & linea KZL talis ut (ducta MFZ ad axem AD normali) ſit MP correſpondenti FZ æqualis; erit ſuperficies producta à rotatione Curvæ AMB circa axem

axem AD, ad ſpatium ADLK, ut Circumferentia Circuli ad ſuum radium.

Hoc etiam unum eſt ex innumeris & præclaris Theorematis viri celeberrimi D. Iſaaci Barrow.

Prob. I.

Superficiem Sphæræ determinare.

SIT AMB Semicirculus à cujus rotatione data ſphæra producitur: & deſignet r radium & c circumferentiam cujuslibet circuli; & ſit AB (diameter Semicirculi AMB) $= 2d$. jam quoniam omnes lineæ Circulo perpendiculares MP perveniunt ad Circuli centrum P; ideo erit KZL, parallelogrammum rectangulum cujus longitudo diameter AB & altitudo AK $= d$ radius Semicirculi AMB; unde AL $= 2d^2$; (per literam s ubiq; deſigno ſuperficiem Curvam;) ideo per Theorem tertium $s. 2d^2 :: c. r.$ unde $s = \frac{2d^2c}{r}$; vel ponendo $r = d$ erit $s = 2dc$; ac propterea Superficies Sphæræ æquatur rectangulo cujus Longitudo eſt Circumferentia & Latitudo Diameter Circuli in Sphæra maximi. Fig. 18.

Notatu dignum arbitror hinc conſequi omnium

 Theo-

Theorematum longe nobiliſſimum quo æternam ſibi famam acquiſivit Geometrarum Princeps *Archimedes*; Quod ſcilicet ſuperficies Sphæræ ſit æqualis quatuor maximis in eâ Circulis. Sit enim $Q =$ maximo in Sphæra Circulo; at qui $Q = \frac{dc}{2}$ ut ab *Archimede* demonſtratum eſt; Ergo $2Q = dc$, & $4Q = 2dc$; ſed jam inventum eſt $s = 2dc$; Ergo $4Q = s$. Quod erat demonſtrandum.

PROB. II.

Superficiem Conoidis Parabolici determinare.

ESto r latus rectum Parabolæ AMB à cujus rotatione conoides producitur, ſit axis AD, vertex A & vocetur AF, y; FM x; per methodum aliquam tangentium invenietur $PMq = \frac{1}{4}r^2 + ry$; vel ponendo $FZ = z$, quia ſupponitur $PM = FZ$, aut $\frac{1}{4}rx + ry = Z^2$ quæ eſt æquatio ad parabolam cujus axis idem eſt cum axe parabolæ datæ AMB; cujus vertex eſt C, exiſtente $AC = \frac{1}{4}r$; & latus illius rectum etiam r, invenietur $AKLD = \sqrt{\frac{4}{9}rv^3} - \frac{1}{12}r^2$ exiſtente $CD = v$; ſed $s. \sqrt{\frac{4}{9}rv^3} - \frac{1}{12}r^2 :: c. r.$ per Theorema 3. Ergo $s = \sqrt{\frac{4c^2v^3}{9r}} - \frac{1}{12}rc$.

In

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In hunc modum menſurantur non modo ſuperficies Conoidis Hyperbolici, & Sphæroidis, ſed Quævis alia Curva ſuperficies quæ generatur à rotatione lineæ Curvæ & hæc duo exempla ſatis oſtendunt quomodo eadem Methodus ad omnes alias ſuperficies Curvas ſit Applicanda.

ANIM-

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ANIMADVERSIO

In Methodum Figuras dimetiendi,

A clariſſimo Quodam Germano editam in Actis Eruditorum Lipſiæ publicatis.

METHODUM hanc propoſuerat Doctiſſimus illius Author Anno 1683, Menſe *Octob.* quam adeo perfectam credebat; ut vel Quadraturam Figuræ, vel ejuſdem Quadraturæ impoſſibilitatem determinaret; & ex ea Circuli & Hyperbolæ Quadraturam Geometricam impoſſibilem eſſe concluſerat. Poſtea vero perſpexit clariſſimus vir tanta perfectione præditam non eſſe, ut exinde Circuli, Hyperbolæ aut alterius Figuræ Quadraturæ impoſſibilitas probari poſſit, ut ingenue ipſe fatetur in iiſdem actis Anni ſequentis, ubi ait ſe amore veritatis coactum hoc unum monere. Unde exiſtimat quaſdam eſſe Figuras quæ indefinitæ Quadraturæ non ſunt capaces & exemplum Figuræa ſcribit in qua ſuccedere ait Quadraturam ſpecialem ſine generali: in hoc tamen hallucinatus eſt clariſ. vir quod ex ſua Methodo Figuram aliquam, Quadraturam indefinitam recuſare concluſit; priuſquam demonſtraſſet

Metho-

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Methodum ſuam ad omnes figuras indefinitè quadrabiles extendere; quod demonſtratu eſt impoſſibile, cum unam è milleſimis non comprehendat; ut poſtea patebit. Dantur enim infinitæ figuræ indefinitæ quadrabiles, quæ nullo modo per iſtam Methodum ſunt Quadrabiles; & quarum exempla poſtea apponam; Et ut non modo errorem ſed erroris fontem detegam, neceſſe videtur breve illius Methodi compendium adjungere.

Adhibet æquationes Curvarum Generales, quarum unaquæque omnes Curvas ejuſdem gradus exprimere exiſtimat: Et talis Curvæ generalis conſideratæ tanquam Quadratricis quærit Quadrandam Generalem. Et oblatæ Quadrandæ ſpecialis æquationem comparat cum aliqua ex formulis generalibus Quadrandaram naturam exprimentibus; unde deducit Quadratricem ſpecialem Quadrandæ ſpeciali convenientem; exemplo res erit manifeſta.

Sit ABC figura, rectis AC, CB & Curva AB comprehenſa, ſitq; ACDE = ABC, AGF = AGHL, & idem concipiatur ubique, proveniet hinc Curva aliqua AHD, quam Quadratricem appellat, quia illius ope Quadratur area ABC, jam æquationem aſſumit ad Quadratricem generalem AHD, & ex ea deducit Quadrandam generalem ABC: ut ſi notentur abſciſſæ AG, AC per x, & ordinatæ Quadratricis CD, GH per y, & deniq; ordinatæ in Quadranda per z, ponitq; æquationem ad Quadratricem generalem in qua ordinata x eſt duarum dimenſio-

menſionum hujuſmodi, $\left.\begin{array}{r} by^2 + cay + ea^2 \\ + xy + fax \\ + gx \end{array}\right\} = 0$

ex qua deducit æquationem ad Quadrandam generalem, in qua ordinata z eſt etiam duarum dimenſionum,

$$\begin{array}{l} bz^2 + caz + ea^2 \\ \quad + 2dxz + 2fax + \\ \qquad + 4gx^2 \end{array} \sqrt{\begin{array}{l} d^2e + c^2g + f^2b - cdf - 4beg + a^2x^2 \\ 4bea^2 + 4bfax + 4bgx^2 \\ - c^2a^2 - 2cdax - d^2x^2 \end{array}} = 0$$

Et ſimiliter pro reliquis Quadratricibus generalibus Quadrandas generales inveſtigat. Proponatur jam Figura aliqua Quadranda ſpecialis ABC, & exprimatur natura Curvæ AFB hac æquatione $Z^2 = \frac{9aax - 12axx + 4x^3}{2a - x}$; hanc æquationem comparat cum æquatione generalis Quadrandæ jam poſitæ, (quia ordinata z in Quadranda ſpeciali ad duas tantum dimenſiones aſcendit) nempe ſingulos hujus cum ſingulis illius terminis (ubi x eandem utrobiq; compoſitronem obtinet) eritque ex hac comparatione $c, d, e = 0$, & $b = \frac{1}{2}$, $f = -1$ ac $g = \frac{1}{2}$; & hos valores ſubſtituit in æquatione ad Quadratricem ſupra poſitam in qua ordinatæ x eſt duarum dimenſionum, (quia hic ordinata z ad duas quoque dimenſiones aſcendit) eritque $\frac{y^2}{2} - ax + \frac{x^2}{2} = 0$, ſeu $y^2 = 2ax - x^2$, proprietas Quadratricis ſpecialis AHD in qua AGF = AGHL adeoq; habetur Figuræ propoſitæ Quadratura.

In eo tamen latet ratiocinationis & ipſius Methodi defectus, quod omnes Curvas in quibus z ad duas dimenſiones (nec ultra) aſcendit comparet cum una & eâdem

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eadem Quadranda generali, in quibus z non ultra duas dimenſiones aſcendit; & quod concludat Figuram non eſſe indefinite Quadrabilem ſi hæc comparatio Quadratricem non determinet. Infinitæ enim ſunt Quadrandæ generales (ex ipſius etiam Methodo deducibiles) in quibus z non ultra duas dimenſiones aſcendit, & non nunquam æquatio Curvæ propoſitæ, cum prima, ſecunda, tertia, &c. comparata Quadratricem non habebit, & tamen comparatio cum Milleſima quadratricem determinabit. Si enim ab æquatione tertia (quam poſuit pro Quadratrice generali in qua x eſſet triuin dimenſionum primum terminum $by^3 + dxy^3$ auferat, ex reliquo Quadrandam generalem deducere poteſt, in qua z non ultra duas dimenſiones aſcendit, & quæ Quadratricem determinet, cum illâ quam ille ſtatuit generalem non ſuccedit: Et ſic ex æquatione quarta, quinta (quas ille poneret pro Quadratricibus altiorum graduum) &c. ablatis iis terminis in quibus y ultra duas dimenſiones aſcendit, ex reliquo habere poteſt Quadrandæ generalis æquatio, quæ Quadratricem determinabit, cum nec ejus Quadranda, nec illa quam dixi eſſe deducibilem ex æquatione tertia, determinare poteſt, ita ut caſu non arte incidimus in Quadrandam Generalem requiſitam. Sed quia dixi iſtas æquationes ad Quadrandas generales ex ipſius Methodo eſſe deducibiles; volo hic paucis oſtendere, Quomodo clariſſ. hic vir æquationes ad Quadrandas generales invenerit, vel ſaltem facile invenire potuiſſet.

 Ex

Ex *Prob.* 22. conſtat, quomodo data æquatione ad Curvam aliquam A H D, alia curva A F B ſit invenienda cujus area A G F æquatur rectangulo comprehenſo ſub ordinatâ G A & abſciſſa A G; id eſt Quomodo data Quadratrice, invenienda ſit Quadranda; adeoq; aſſumpta æquatione ad Quadratricem generalem (qualem hic ſub initio aſcripſi) proveniet æquatio ad Quadrandam generalem. Jamq; exemplum unum aut alterum Figuræ hic aſcribam, in qua Quadratrix ſecundum hanc Methodum eſt impoſſibilis, & tamen alio modo determinabilis. Sit Æquatio naturam curvæ A F B exprimens $z^2 = \overline{\frac{m^2+x}{p}}^{\,2} + x^2$ in qua *x* denotat abſciſſas A C, A G, & *z* ordinatas B C, G F, *m* & *p* quantitates datas & peterminatas; jam ſi Quadranda ſit Area A G F, comparanda eſt hæc æquatio cum æquatione ad Quadrandam generalem jam tradita, quia in hac propoſita æquatione *z* ad duas dimenſiones aſcendit; ſed manifeſtum eſt comparationem non ſuccedere (ut ipſe alibi argumentatur) ſi vel ſolus numerator fractionis utrobique exiſtens comparetur, deberet enim $m^2 + x^2$ coincidere cum $\overline{-d^2e + ag + bf_2 - cdf - 4beg + a^2}$, indeterminatum cum determinato, quod fieri nequit, itaq; figura hoc modo Quadratricem non habet, & tamen ipſa hæc figura eſt indefinite quadrabilis, ſcil. AGF $= \sqrt{\frac{n^6 + 3n^4x^2 + 3n^2x^4 + x^6}{9pp}}$ Et non una tantum ſed infinitæ poſſunt inveniri Figuræ indefinitè Quadrabiles, quarum

Fig. 20.

Fig. 20.

Quadra-

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Quadratrices hoc modo ſunt impoſſibiles per *Prob:* 23.

Fig. 20. Definiatur AHD hac æquatione $x^9 = a^7 y^2$, & per *Prob.* 23. inveniatur curva AFB cujus Area AGF æquetur rectangulo contento ſub ordinata GH & data qualibet recta puta (a), & definietur AFB hac æquatione $z^2 = \frac{81x^7}{4a^5}$; jamq; ſi Quadranda ſit area AGF ſecundum hanc Methodum, comparanda eſt hæc æquatio cum æquatione ad Quadrandam generalem jam tradita, quia in propoſita æquatione z non ultra duas dimenſiones aſcendit; ſed comparatio eſt impoſſibilis, quia in propoſita Curva x ad ſeptimam poteſtatem aſcendit; & in ejus æquatione ad Quadrandam generalem ultra quartam aſcendere non poteſt; ſed terminus in quo x eſt ſeptimæ, non poteſt comparari cum termino in quo x eſt quartæ poteſtatis; nam ſecundum ipſius Regulam, comparatio eſt ſic inſtituenda ut x utrobiq; eandem obtineat compoſitionem; adeoq; Quadratrix hoc modo haberi non poteſt, & tamen Quadratricem habet AHD definita hac æquatione $x^9 = a^7 y^2$, in qua GH $+ a =$ AGF; id eſt $-\sqrt{\frac{x^9}{a^5}} =$ AGF. Unde abunde conſtat hanc Methodum omnes Figuras indefinite Quadrabiles non comprehendere; & infinitas poſſe inveniri quarum Areæ hoc modo non ſunt quadrabiles; aſſumatur enim quælibet æquatio in qua z non ultra duas, & x non infra quatuor dimenſiones reperitur, & habetur æquatio naturam Curvæ exprimens cujus area per hanc Methodum non ſunt quadrabiles; ut in his exemplis z^2

$= \frac{x^8}{a^7}$,

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$=\frac{x^{9}}{a^{7}}$, $z^{2}=\frac{x^{11}}{a^{9}}$, $z^{2}=\frac{x^{13}}{a^{11}}$, &c. quæ ſunt æquationes Curvarum naturas definientes quarum Areæ facile determinantur & tamen nullo modo per hanc Methodum poſſunt inveniri. Sed nolo in hac materia ulterius digredi ſperans clariſſ. virum boni conſulturum quicquid dixerim; quia præcipua ratio quæ me impulit ut hæc ſcriberem, non alia eſſet, quam ut (errores ejus oſtendendo) illum extimulem ad publicanda illa, quibus Geometriam in immenſum ultra terminos à Vieta & Carteſio poſitos ſe promovere poſſe aſſeruit.

FINIS.

ERRATA.

PAg. 4. lin. 20. pro $\frac{ny}{2}$ lege $\frac{nry}{2}$; pag. 5. lin. 2. pro $\frac{nry}{4}$ lege $\frac{nry}{2}$. pag. 6. lin. 12. pro $\frac{3nry^{3}}{2a}$ lege $\frac{3nry^{2}}{2a}$, ibid. lin. 20. pro x lege x^{2}. pag. 14. lin. 11. pro $r\frac{y^{2}}{2r}$ lege $r-\frac{y^{2}}{2r}$ ibid. lin. 17. pro $m=3$ lege $m=\frac{2}{3}$. pag. 19. lin. 14. pro $d-y^{3}$. lege $d^{3}-y^{3}$ pag. 25. lin. 16. pro x lege x^{2}. pag. 28. lin. 3. pro $\sqrt{\frac{25}{16a^{3}}}$ lege $\sqrt{\frac{25}{64a^{3}}}$.

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